Finding the Domain and Range of Functions

To find the domain of a function...

Start by letting the domain be all real numbers. Then remove any values of x which will cause trouble. This means you must make sure that no values in the domain main will lead to ...

1) division by 0
2) even root of a negative number

Find the domain of the following functions.

$f(x) = \sqrt{5x-2}$

Check ① division by 0. Not a problem

② even root.

$5x - 2 \geq 0 \Rightarrow 5x \geq 2 \Rightarrow x \geq \frac{2}{5}$

$D = \{x \mid x \geq \frac{2}{5}\}$

$g(x) = \dfrac{7x}{2x^2 - x - 3}$

Check ① division by 0

$2x^2 - x - 3 \neq 0$

$(2x-3)(x+1) \neq 0$

$2x - 3 \neq 0 \quad x + 1 \neq 0$

so $x \neq \frac{3}{2}$ and $x = -1$

② even root. Not a problem

$D = \{x \mid x \neq 1 \text{ and } x \neq \frac{3}{2}\}$

$h(x) = \dfrac{\sqrt{x+2}}{x^2 - 9}$

check ① division by 0

$x^2 - 9 \neq 0$

$(x-3)(x+3) \neq 0$

$x - 3 \neq 0 \quad x + 3 \neq 0$

$x \neq 3$ or $x \neq -3$

② even root

$x + 2 \geq 0$

$x \geq -2$

Take both into consideration

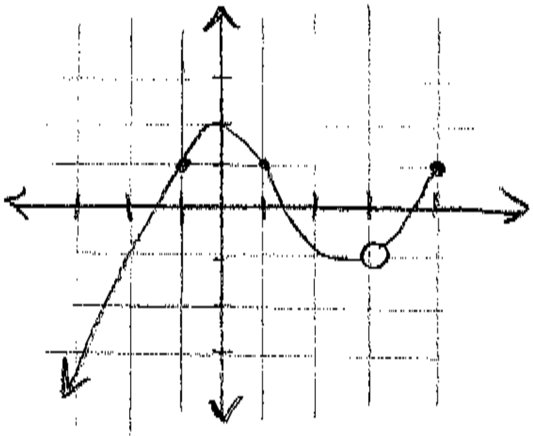
$D = \{x \mid x \geq -2 \text{ and } x \neq 3\}$

$k(x) = \dfrac{\sqrt[3]{x}}{x^2 + 1}$

Domain = { all real numbers } Why? No even roots, x^2+1 is always > 0 so denominator can never be 0.

Answer the following questions for each graph of a function.

What is the domain? What is the range? What is $f(3)$? For what value(s) of x is $f(x) = 1$?



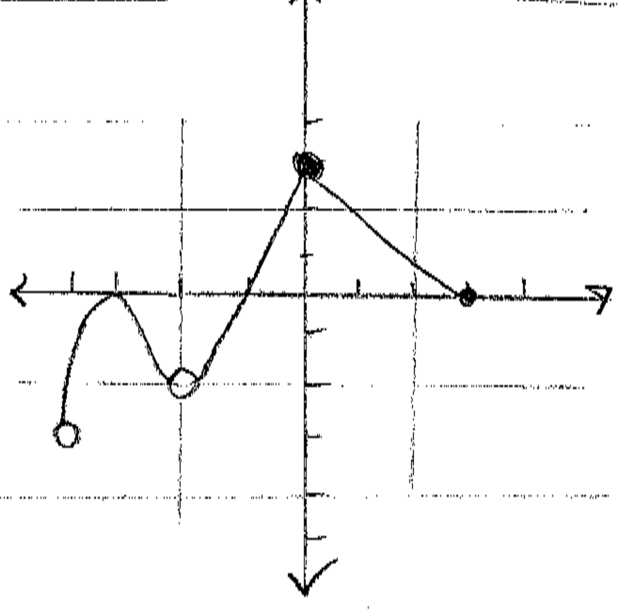
$D = \{x \mid x < 3 \text{ or } 3 < x \leq 4\}$

$R = \{y \mid y \leq 2\}$

$f(3)$: is undefined

$f(x) = 1$ for $x = -1, 1, 4$

For domain and range,
scan up and down x-
and y-axis to see what's included.



$D = \{x \mid -4 < x < -2 \text{ or } -2 < x \leq 3\}$

$R = \{y \mid -3 < y < -2 \text{ or } -2 < y \leq 3\}$

$f(3) = 0$

$f(x) = 1$ for $x = 2, -\frac{2}{3}$